DCMT101

Reg. No. | | | | | | | | |

I Semester B.Sc. Degree Examination, April - 2023

MATHEMATICS

Algebra - I and Calculus - I

Paper : I

(NEP Core Scheme)

Time : 2½ Hours **Maximum Marks : 60**

Instructions to Candidates:

Answer all questions.

I. Answer **any SIX** questions. **(6×2=12)**

1. Find the value of k for which the system of equations $2x-y+2z=0$; $3x+y-z=0$; $kx-2y+z=0$ has non trivial solutions.

2. Find the Eigen values of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

3. Find the n[th] derivative of $e^{2x}\sin 3x$.

4. If $u=x^2+y^2-3xy$ find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

5. Find the value of k such that the function $f(x)=\begin{cases} kx^2 & if\ x>2 \\ 8 & if\ x\le 2 \end{cases}$ is continuous at $x=2$.

6. State Euler's theorem for the homogeneous function.

7. Verify Rolle's theorem for the function $f(x)=8x-x^2$ in [2,6].

8. Evaluate : $\lim_{x\to 0}\frac{1-\cos x}{x^2}$, using L'Hospital rule.

II. Answer **any THREE** of the following. **(3×4=12)**

9. Find the rank of matrix $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by reducing to echelon form.

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10. Solve completely the system of equations $x+2y-4z=0$; $3x-y+2z=0$ and $5x-3y+z=0$.

11. Find the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} 5 & -1 \\ 4 & 9 \end{bmatrix}$.

12. State and prove Caley - Hamilton theorem.

13. Using Caley - Hamilton theorem, find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$.

III. Answer any THREE questions. (3×4=12)

14. Evaluate $\lim_{x\to 0} \frac{e^{1/x}}{1+e^{1/x}}$.

15. Examine the differentiability of the function $f(x)=\begin{cases} x^2-1 & for\ x\geq 1 \\ 1-x & for\ x<1 \end{cases}$ at $x=1$.

16. Prove that a function which is continuous on a closed interval is bounded.

17. Find the n[th] derivative of $\cos x \cos 2x \cos 3x$.

18. If $y=\sin^{-1}x$ prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$.

IV. Answer any THREE questions. (3×4=12)

19. State and prove intermediate value theorem.

20. State and prove Rolle's theorem.

21. Find the Taylor expansion of $f(x)=\cos x$ about the point $x=\frac{\pi}{2}$ upto 4[th] degree terms.

22: Expand $\log(1+x)$ upto the term containing x^4 using Maclaurin's series.

23. Evaluate : $\lim_{x\to\frac{\pi}{2}} (\sin x)^{\tan x}$ using L'Hospital rule.

V. Answer any THREE questions. (3×4=12)

24. If $z=\sin(ax+y)+\cos(ax-y)$ prove that $\frac{\partial^2 z}{\partial x^2}=a^2.\frac{\partial^2 z}{\partial y^2}$.

25. If $z=x^2+y^2$ where $x=e^t\cos t$, $y=e^t\sin t$ find $\frac{dz}{dt}$.

26. Verify Euler's theorem for the function $u=ax^2+2hxy+by^2$.

27. Expand $e^x\sin y$ in powers of x and y upto second degree terms.

28. Find the extreme values of the function $f(x,y)=x^3+y^3-3x-12y+20$.